Secrecy Performance Analysis on UAV Down-Link Broadcasting with a Full Duplex Receiver

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Outline

System Model

In a rural subregion, a UAV-enabled downlink wireless transmission scenario is considered within a three-dimensional (3D) Cartesian cylinder coordinate system where its radius is denoted as D, in which a UAV (Alice) transmits wireless messages to a legitimate receiver (Bob) in the presence of a passive eavesdropper (Eve). Besides, Bob is assumed to possess dual antennas, while Alice and Eve equip N_{A} antennas and a single antenna, respectively. Specifically, Bob is working in the FD mode with one antenna for AN emitting and the other for simultaneous information reception. Due to UAV's high operational altitude and LoS-dominated air-to-ground (A2G) links, it becomes easier for ground-based malicious party to eavesdrop wireless signals emitted from UAVs, undoubtedly highlighting the importance of secrecy communications. To help achieve secure wireless transmissions, FD-BBJ strategy is adopted to allow Bob to generate AN for interfering with Eve's wiretap.

Main Contribution

- With practical assumption on imperfect self-interference cancellation (SIC), closed-form cumulative distribution function (CDF) and probability density function (PDF) expressions of received signal-to-interference-and-noise ratios (SINRs) at the legitimate receiver and the eavesdropper are derived, respectively. Then, closed-form expression of the approximate ergodic achievable secrecy rate (EASR) and the compact expression of the secrecy outage probability (SOP) are calculated.
- To gain more insights, asymptotic secrecy performance of extreme total system transmit power is analysed, after deriving closed-form expression of the asymptotic EASR and compact expression of the asymptotic SOP.
- Numerical results are provided to validate correctness of the derived analytical formulas, showcase effectiveness of FD-BBJ solution for enhancing secrecy transmission of UAV-aided down-link broadcasting channels, and track impacts of various system parameters, e.g., transmit power, on the evaluated metrics.

System Model

The received signal at Bob and Eve can be given by

$$y_{\rm B} = \sqrt{P_{\rm A} 10^{\frac{-\Psi_{\rm AB}}{10}}} \mathbf{h}_{\rm AB} \mathbf{s} + \sqrt{\rho P_{\rm B}} h_{\rm BB} v + n_{\rm B}, \quad (1)$$

$$y_{\rm E} = \sqrt{P_{\rm A} 10^{\frac{-\Psi_{\rm AE}}{10}}} \mathbf{h}_{\rm AE} \mathbf{s} + \sqrt{P_{\rm B} d_{\rm BE}^{-\eta_{\rm BE}}} h_{\rm BE} v + n_{\rm E}, \quad (2)$$

where $\mathbf{s} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}/N_{\rm A})$ is the signal emitted from Alice, $\mathbf{I}$ denotes the $N_{\rm A} \times N_{\rm A}$ unit matrix and $v \sim \mathcal{CN}(0,1)$ is the AN signal used to interfere the eavesdropper. Moreover, $\rho \in [0,1]$ is a normalized coefficient which represents the degree of SIC, where $\rho = 1$ means that there is no SIC applied at Bob, while $\rho = 0$ signifies the perfect SIC and $\rho \in (0,1)$ denotes the imperfect SIC. Besides, $n_{\rm B}$ and $n_{\rm E}$ are the addictive white Gaussian noises (AWGNs) at Bob and Eve with zero mean and variances $\sigma_{\rm B}^2$ and $\sigma_{\rm E}^2$, respectively.

SNIR Formulation

Moreover, the path loss model on the sub-6 GHz band is considered to characterize the large-scale fading for A2G wireless links, given by $\Psi_{\mathrm{A}j}(\mathrm{dB}) = 20\lg(d_{\mathrm{A}j}) + 20\lg(\varpi) - 147.55$, where $d_{\mathrm{A}j}$ denotes the Euclidean distance between Alice and $j \in (\mathrm{B}, \mathrm{E})$, and ϖ represents the carrier frequency.

From (1) and (2), the received SINRs at Bob and Eve can be calculated as

$$\gamma_{\mathrm{B}} = \frac{P_{\mathrm{A}} 10^{\frac{-\Psi_{\mathrm{AB}}}{10}} \|\mathbf{h}_{\mathrm{AB}}\|^2}{\rho N_{\mathrm{A}} P_{\mathrm{B}} |h_{\mathrm{BB}}|^2 + N_{\mathrm{A}} \sigma_{\mathrm{B}}^2}, \tag{3}$$

$$\gamma_{\mathrm{E}} = \frac{P_{\mathrm{A}} 10^{\frac{-\Psi_{\mathrm{AE}}}{10}} \|\mathbf{h}_{\mathrm{AE}}\|^2}{N_{\mathrm{A}} P_{\mathrm{B}} d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}} |h_{\mathrm{BE}}|^2 + N_{\mathrm{A}} \sigma_{\mathrm{E}}^2}. \tag{4}$$

The Considered Performance Metrics

In the considered model, the secrecy capacity can be expressed as $C_\mathrm{S} = [C_\mathrm{B} - C_\mathrm{E}]^+$, where $[x]^+ \triangleq \max\{0, x\}$, and $C_\mathrm{B} = \log_2(1+\gamma_\mathrm{B})$ and $C_\mathrm{E} = \log_2(1+\gamma_\mathrm{E})$ are mutual information of the legitimate and eavesdropping channels, respectively. The ergodic secrecy capacity is defined as the rate below which any average secure communication rate is achievable and formulated under block fading channels as

$$\begin{aligned}\mathbb{E}[C_\mathrm{S}] &= \int_0^\infty \int_0^\infty [C_\mathrm{B} - C_\mathrm{E}]^+ f(\gamma_\mathrm{B}) f(\gamma_\mathrm{E}) \, d\gamma_\mathrm{B} d\gamma_\mathrm{E} \\ &= \mathbb{E}\left[[C_\mathrm{B} - C_\mathrm{E}]^+\right]. \end{aligned} \tag{5}$$

The Considered Performance Metrics

However, the exact evaluation of (5) appears to be intractable for our considered system. Alternatively, we focus our analysis on a lower bound of (5), expressed as

$$\mathbb{E}\left[C_{\mathrm{S}}\right] \geq \left[\mathbb{E}\left[C_{\mathrm{B}}\right] - \mathbb{E}\left[C_{\mathrm{E}}\right]\right]^{+} \triangleq \overline{C}_{\mathrm{S}}, \tag{6}$$

which is known as ergodic achievable secrecy rate (EASR).

Besides, the secrecy outage probability (SOP) is defined as the probability that the achievable secrecy rate is less than a given secrecy transmission rate R_{th}, below which secure transmission is not guaranteed. In our considered system, the SOP can be formulated as

$$P_{out}\left(R_{th}\right) = \Pr\left(C_S \leq R_{th}\right) = \Pr\left(\frac{1+\gamma_{\mathrm{B}}}{1+\gamma_{\mathrm{E}}} \leq 2^{R_{th}}\right). \tag{7}$$

The Statistics of γ_{B}

$$\text{CDF}: F_{\gamma_{\mathrm{B}}}(x) = 1 - \sum_{u=0}^{m_{\mathrm{B}}N_{\mathrm{A}}-1}\sum_{v=0}^{u}\binom{u}{v}(N_{\mathrm{A}}m_{\mathrm{B}}x)^u\, v!\,\sigma_{\mathrm{B}}^{2(u-v)} \frac{P^{v-u}\left(\alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}}\Omega_{\mathrm{AB}}\right)^{v+1-u}(\rho(1-\alpha)\Omega_{\mathrm{BB}})^{v}}{u!\left(\rho(1-\alpha)N_{\mathrm{A}}m_{\mathrm{B}}\Omega_{\mathrm{BB}}x + \alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}}\Omega_{\mathrm{AB}}\right)^{v+1}} \tag{8}$$

$$\text{PDF}: f_{\gamma_{\mathrm{B}}}(x) = \sum_{u=0}^{m_{\mathrm{B}}N_{\mathrm{A}}-1}\sum_{v=0}^{u}\binom{u}{v} v!\left(\alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}}\Omega_{\mathrm{AB}}\right)^{v+1-u} P^{v-u} \frac{\sigma_{\mathrm{B}}^{2(u-v)}(\rho(1-\alpha)\Omega_{\mathrm{BB}})^{v}(N_{\mathrm{A}}m_{\mathrm{B}})^{u}x^{u-1}}{u!\left(\rho(1-\alpha)N_{\mathrm{A}}m_{\mathrm{B}}\Omega_{\mathrm{BB}}x + \alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}}\Omega_{\mathrm{AB}}\right)^{v+2}} \times\left[\alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}}\Omega_{\mathrm{AB}}u + \rho(1-\alpha)N_{\mathrm{A}}m_{\mathrm{B}}\Omega_{\mathrm{BB}}(u-v-1)x\right] \tag{9}$$

The Statistics of γ_{E}

$$\text{CDF}: F_{\gamma_{\mathrm{E}}}(x) = 1 - \sum_{p=0}^{m_E N_{\mathrm{A}}-1} \sum_{q=0}^{p} \binom{p}{q} (N_{\mathrm{A}} m_{\mathrm{E}} x)^p q! \sigma_{\mathrm{E}}^{2(p-q)} \frac{P^{q-p}\left(\alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}} \Omega_{\mathrm{AE}}\right)^{q+1-p} \left((1-\alpha) d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}} \Omega_{\mathrm{BE}}\right)^q}{p! \left((1-\alpha) N_{\mathrm{A}} m_{\mathrm{E}} d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}} \Omega_{\mathrm{BE}} x + \alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}} \Omega_{\mathrm{AE}}\right)^{q+1}} \tag{10}$$

$$\text{PDF}: f_{\gamma_{\mathrm{E}}}(x) = \sum_{p=0}^{m_{\mathrm{E}} N_{\mathrm{A}}-1} \sum_{q=0}^{p} \binom{p}{q} q! \left(\alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}} \Omega_{\mathrm{AE}}\right)^{q+1-p} \frac{P^{q-p} \sigma_{\mathrm{E}}^{2(p-q)} \left((1-\alpha) d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}} \Omega_{\mathrm{BE}}\right)^q (N_{\mathrm{A}} m_{\mathrm{E}})^p x^{p-1}}{p! \left(N_{\mathrm{A}} m_{\mathrm{B}} (1-\alpha) d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}} \Omega_{\mathrm{BE}} x + \alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}} \Omega_{\mathrm{AE}}\right)^{q+2}} \times \left[\alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}} \Omega_{\mathrm{AE}} p + (1-\alpha) N_{\mathrm{A}} m_{\mathrm{E}} d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}} \Omega_{\mathrm{BE}} (p-q-1) x\right] \tag{11}$$

EASR Analysis

Theorem

The closed-form expression for the approximate ergodic achievable rate of the legitimate channel, i.e., $\mathbb{E}[C_{\mathrm{B}}]$*, can be calculated as*

$$\mathbb{E}[C_{\mathrm{B}}] \approx \frac{1}{\ln 2} \sum_{\varpi=1}^{\vartheta} \omega_{\varpi} \Phi(z_{\varpi}), \tag{12}$$

where z_{ϖ} $(\varpi = 0, 1, \ldots, \vartheta)$ *is the* ϖ*-th root of the Laguerre polynomial* $\mathcal{L}_{\vartheta}(z)$ *and* ω_{ϖ} *which does not depend on* $\Phi(z)$ *is the* ϖ*-th weight given by*

$$\omega_{\varpi} = \frac{z_{\varpi}}{\left[(\vartheta + 1)\mathcal{L}_{\vartheta+1}(z_{\varpi})\right]^2}. \tag{13}$$

ϑ *denotes the number of points used to approximate the integral. It is meaningful to note that both* z_{ϖ} *and* ω_{ϖ} *can be calculated efficiently using the algorithm provided in [R1].*

[R1] G. W. Recktenwald, Numerical Methods with MATLAB: Implementation and Application, 2000.

EASR Analysis

Proof.

We can formulate $\mathbb{E}\left[C_{\mathrm{B}}\right]$ as

$$\begin{aligned}\mathbb{E}\left[C_{\mathrm{B}}\right] &= \frac{1}{\ln 2}\mathbb{E}\left[\ln\left(1+\gamma_{\mathrm{B}}\right)\right] \\ &= \frac{1}{\ln 2}\int_{0}^{\infty}\frac{1-F_{\gamma_{\mathrm{B}}}\left(x\right)}{1+x}dx.\end{aligned} \tag{14}$$

Invoking (8) into (14), we can get the semi-closed-form expression of $\mathbb{E}\left[C_{\mathrm{B}}\right]$ as

$$\mathbb{E}\left[C_{\mathrm{B}}\right] = \frac{1}{\ln 2}\int_{0}^{+\infty} e^{-x}\Phi\left(x\right)dx. \tag{15}$$

EASR Analysis

where

$$\Phi(x) = \sum_{u=0}^{m_{\mathrm{B}} N_{\mathrm{A}}-1} \sum_{v=0}^{u} \binom{u}{v} \left(N_{\mathrm{A}} m_{\mathrm{B}} x\right)^{u} v! \sigma_{\mathrm{B}}^{2(u-v)} \times \frac{P^{v-u} \left(\rho(1-\alpha)\Omega_{\mathrm{BB}}\right)^{v}}{\left(\rho(1-\alpha) N_{\mathrm{A}} m_{\mathrm{B}} \Omega_{\mathrm{BB}} x + \alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}} \Omega_{\mathrm{AB}}\right)^{v+1}} \times \frac{e^{x} \left(\alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}} \Omega_{\mathrm{AB}}\right)^{v+1-u}}{u!(1+x)}. \tag{16}$$

The integral in (15) can not be derived to a closed-form. As such, we resort to adopt the Gauss-Laguerre Quadrature (GLQ) method to approach the integral with finite summation. Then, (12) can be obtained.

EASR Analysis

Theorem

Closed-form expression of the approximate ergodic achievable rate of the eavesdropping channel, i.e., $\mathbb{E}[C_{\mathrm{E}}]$*, can be derived as*

$$\mathbb{E}[C_{\mathrm{E}}] \approx \frac{1}{\ln 2} \sum_{\varpi=1}^{\vartheta} \omega_{\varpi} \mathcal{H}(z_{\varpi}), \tag{17}$$

where

$$\mathcal{H}(x) = \sum_{p=0}^{m_E N_{\mathrm{A}}-1} \sum_{q=0}^{p} \binom{p}{q} (N_{\mathrm{A}} m_{\mathrm{E}} x)^p q! \sigma_{\mathrm{E}}^{2(p-q)} \times \frac{P^{q-p}\left(\left(1-\alpha\right) d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}} \Omega_{\mathrm{BE}}\right)^q}{\left(\left(1-\alpha\right) N_{\mathrm{A}} m_{\mathrm{E}} d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}} \Omega_{\mathrm{BE}} x + \alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}} \Omega_{\mathrm{AE}}\right)^{q+1}} \times \frac{e^x \left(\alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}} \Omega_{\mathrm{AE}}\right)^{q+1-p}}{p!\,(1+x)}. \tag{18}$$

EASR Analysis

From *Theorem 1*, *Theorem 2* and (6), Closed-form expression of the approximate EASR can be formulated as

$$\overline{C}_{\mathrm{S}} \approx \frac{1}{\ln 2}\left[\sum_{\varpi=1}^{\vartheta} \omega_{\varpi}\Phi\left(z_{\varpi}\right) - \sum_{\varpi=1}^{\vartheta} \omega_{\varpi}\mathcal{H}\left(z_{\varpi}\right)\right]^{+}. \tag{19}$$

SOP Analysis

Invoking (7), (8) and (11) and after some mathematical manipulations, SOP can be derived, where step (a) stands due to the binomial expansion, i.e., $\left[2^{R_{th}}\left(1+x\right)-1\right]^{u}=\sum_{\varepsilon=0}^{u}\left(2^{R_{th}}x\right)^{\varepsilon}\left(2^{R_{th}}-1\right)^{u-\varepsilon}$.

$$
\begin{aligned}
P_{out} = 1 - &\sum_{u=0}^{m_{\mathrm{B}}N_{\mathrm{A}}-1}\sum_{v=0}^{u}\sum_{p=0}^{m_{E}N_{\mathrm{A}}-1}\sum_{q=0}^{p}\binom{u}{v}\binom{p}{q}\frac{v!q!\sigma_{\mathrm{B}}^{2(u-v)}\sigma_{\mathrm{E}}^{2(p-q)}P^{v+q-u-p}N_{\mathrm{A}}^{u+p}m_{\mathrm{B}}^{u}m_{\mathrm{E}}^{p}\alpha^{v+q+2-u-p}\left(1-\alpha\right)^{v+q}}{u!p!} \\
&\times\left(10^{\frac{-\Psi_{\mathrm{AB}}}{10}}\Omega_{\mathrm{AB}}\right)^{v+1-u}\left(\rho\Omega_{\mathrm{BB}}\right)^{v}\left(10^{\frac{-\Psi_{\mathrm{AE}}}{10}}\Omega_{\mathrm{AE}}\right)^{q+1-p}\left(d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}}\Omega_{\mathrm{BE}}\right)^{q} \\
&\times\int_{0}^{+\infty}\frac{\left[2^{R_{th}}\left(1+x\right)-1\right]^{u}}{\left(\rho\left(1-\alpha\right)N_{\mathrm{A}}m_{\mathrm{B}}\Omega_{\mathrm{BB}}\left(2^{R_{th}}\left(1+x\right)-1\right)+\alpha10^{\frac{-\Psi_{\mathrm{AB}}}{10}}\Omega_{\mathrm{AB}}\right)^{v+1}} \\
&\times\frac{\left[\alpha10^{\frac{-\Psi_{\mathrm{AE}}}{10}}\Omega_{\mathrm{AE}}p+\left(1-\alpha\right)N_{\mathrm{A}}m_{\mathrm{E}}d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}}\Omega_{\mathrm{BE}}\left(p-q-1\right)x\right]x^{p-1}}{\left(N_{\mathrm{A}}m_{\mathrm{E}}\left(1-\alpha\right)d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}}\Omega_{\mathrm{BE}}x+\alpha10^{\frac{-\Psi_{\mathrm{AE}}}{10}}\Omega_{\mathrm{AE}}\right)^{q+2}}dx
\end{aligned}
$$

SOP Analysis

$$
\begin{aligned}
&\overset{a}{=} 1 - \sum_{u=0}^{m_{\mathrm{B}} N_{\mathrm{A}}-1} \sum_{v=0}^{u} \sum_{p=0}^{m_E N_{\mathrm{A}}-1} \sum_{q=0}^{p} \sum_{\varepsilon=0}^{u} \binom{u}{v} \binom{p}{q} \frac{\left(10^{\frac{-\Psi_{\mathrm{AB}}}{10}} \Omega_{\mathrm{AB}}\right)^{v+1-u} \left(\rho \Omega_{\mathrm{BB}}\right)^{v} \left(10^{\frac{-\Psi_{\mathrm{AE}}}{10}} \Omega_{\mathrm{AE}}\right)^{q+1-p} \left(d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}} \Omega_{\mathrm{BE}}\right)^{q}}{u!p!} \\
&\quad \times 2^{R_{th}\varepsilon} \left(2^{R_{th}} - 1\right)^{u-\varepsilon} v!q! \sigma_{\mathrm{B}}^{2(u-v)} \sigma_{\mathrm{E}}^{2(p-q)} P^{v+q-u-p} N_{\mathrm{A}}^{u+p} m_{\mathrm{B}}^{u} m_{\mathrm{E}}^{p} \alpha^{v+q+2-u-p} (1-\alpha)^{v+q} \\
&\quad \times \int_0^{+\infty} \frac{\left[\alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}} \Omega_{\mathrm{AE}} p + (1-\alpha) N_{\mathrm{A}} m_{\mathrm{E}} d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}} \Omega_{\mathrm{BE}} (p-q-1) x\right] x^{p+\varepsilon-1}}{\left(\rho (1-\alpha) N_{\mathrm{A}} m_{\mathrm{B}} \Omega_{\mathrm{BB}} \left(2^{R_{th}} (1+x) - 1\right) + \alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}} \Omega_{\mathrm{AB}}\right)^{v+1}} \\
&\quad \times \frac{1}{\left(N_{\mathrm{A}} m_{\mathrm{E}} (1-\alpha) d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}} \Omega_{\mathrm{BE}} x + \alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}} \Omega_{\mathrm{AE}}\right)^{q+2}} dx \qquad (20)
\end{aligned}
$$

Asymptotic Analysis

Closed-form expressions of approximate EASR and the compact SOP expression have been calculated in the last section. To gain simple yet meaningful conclusions and analyse the secure performance of the considered system more effectively, in this section, we will provide analysis for EASR and SOP in the asymptotic case where the total transmit power of the system tends to infinity, i.e., $P \to +\infty$.

Asymptotic EASR Analysis

$$\overline{C}_{\mathrm{S}}^{P\to+\infty}$$

$$= -\sum_{u=0}^{m_{\mathrm{B}}N_{\mathrm{A}}-1} \frac{\alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}}\Omega_{\mathrm{AB}}\left[\rho\left(1-\alpha\right)N_{\mathrm{A}}m_{\mathrm{B}}\Omega_{\mathrm{BB}}\right]^{u}\left[\mathrm{B}\left(\frac{\rho(1-\alpha)N_{\mathrm{A}}m_{\mathrm{B}}\Omega_{\mathrm{BB}}}{\alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}}\Omega_{\mathrm{AB}}},-u,u+1\right)+\pi\csc\left(u\pi\right)\right]}{\left(\alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}}\Omega_{\mathrm{AB}}-\rho\left(1-\alpha\right)N_{\mathrm{A}}m_{\mathrm{B}}\Omega_{\mathrm{BB}}\right)^{u+1}}$$

$$+\sum_{p=0}^{m_{\mathrm{E}}N_{\mathrm{A}}-1} \frac{\alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}}\Omega_{\mathrm{AE}}\left[\left(1-\alpha\right)N_{\mathrm{A}}d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}}m_{\mathrm{E}}\Omega_{\mathrm{BE}}\right]^{p}\left[\mathrm{B}\left(\frac{(1-\alpha)N_{\mathrm{A}}d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}}m_{\mathrm{E}}\Omega_{\mathrm{BE}}}{\alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}}\Omega_{\mathrm{AE}}},-p,p+1\right)+\pi\csc\left(p\pi\right)\right]}{\left(\alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}}\Omega_{\mathrm{AE}}-\left(1-\alpha\right)N_{\mathrm{A}}d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}}m_{\mathrm{E}}\Omega_{\mathrm{BE}}\right)^{p+1}} \tag{21}$$

where $\mathrm{B}\left(\cdot,\cdot,\cdot\right)$ is the incomplete Beta function and $\csc\left(\cdot\right)$ denotes the cosection function.

Asymptotic SOP Analysis

$$
\begin{aligned}
P_{out}^{P\to+\infty} = 1 - &\sum_{u=0}^{m_{\mathrm{B}}N_{\mathrm{A}}-1}\sum_{p=0}^{m_E N_{\mathrm{A}}-1}\sum_{q=0}^{u}\binom{u}{q}\frac{\alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}}\Omega_{\mathrm{AB}}\left[\rho\left(1-\alpha\right)N_{\mathrm{A}}m_{\mathrm{B}}\Omega_{\mathrm{BB}}\right]^{u}2^{qR_{th}}\left(2^{R_{th}}-1\right)^{u-q}}{u!p!}\\
&\times \alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}}\Omega_{\mathrm{AE}}\left[\left(1-\alpha\right)N_{\mathrm{A}}d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}}m_{\mathrm{E}}\Omega_{\mathrm{BE}}\right]^{p}\\
&\times\int_0^{+\infty}\frac{\left[\alpha p 10^{\frac{-\Psi_{\mathrm{AE}}}{10}}\Omega_{\mathrm{AE}}-\left(1-\alpha\right)N_{\mathrm{A}}d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}}m_{\mathrm{E}}\Omega_{\mathrm{BE}}x\right]x^{p+q-1}}{\left(\rho\left(1-\alpha\right)N_{\mathrm{A}}m_{\mathrm{B}}\Omega_{\mathrm{BB}}\left(2^{R_{th}}\left(1+x\right)-1\right)+\alpha 10^{\frac{-\Psi_{\mathrm{AB}}}{10}}\Omega_{\mathrm{AB}}\right)^{u+1}}\\
&\times\frac{1}{\left(N_{\mathrm{A}}m_{\mathrm{E}}\left(1-\alpha\right)d_{\mathrm{BE}}^{-\eta_{\mathrm{BE}}}\Omega_{\mathrm{BE}}x+\alpha 10^{\frac{-\Psi_{\mathrm{AE}}}{10}}\Omega_{\mathrm{AE}}\right)^{p+2}}dx \qquad (22)
\end{aligned}
$$

Simulation Setups

The following shows the numerical results to validate EASR and SOP analyses through Monte Carlo simulation method and then explore the impact of parameters on the considered metrics. EASR and SOP curves are generated from the analytical results of (19) and (20), while the asymptotic curves are plotted as per (21) and (22), respectively. The Monte Carlo simulation points are calculated by taking average over 10^6 random channel realizations. Without loss of generality, we assume unit variance for all involved channel coefficients and AWGNs' variances are given by $\sigma_{\mathrm{B}}^2 = \sigma_{\mathrm{E}}^2 = -60$dBm, while the carrier frequency is fixed at $\varpi = 2$GHz. Path loss exponent for terrestrial transmission is adopted as $\eta_{\mathrm{BE}} = 3$, while Nakagami parameters are considered as $m_{\mathrm{B}} = m_{\mathrm{E}} = 2$. The amount of GLQ points used to approximate the EASR is set as $\vartheta = 24$.

Simulation Figures

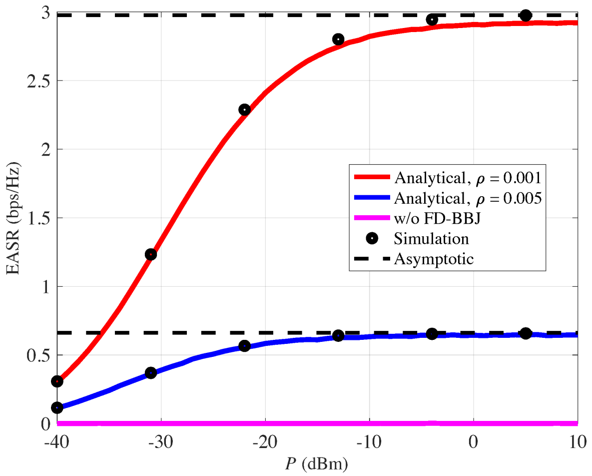

Figure 2: EASR versus P for various ρ.

Simulation Figures

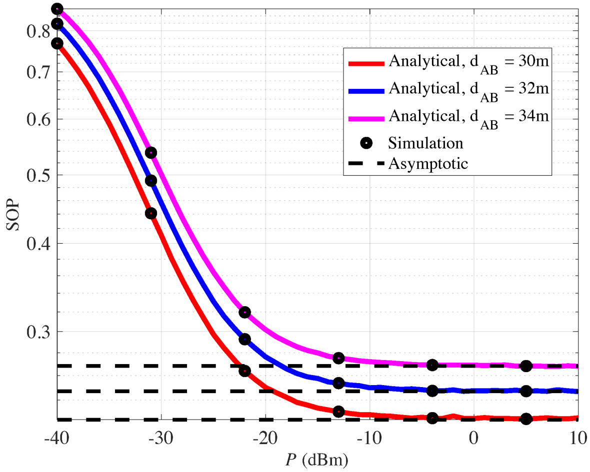

Figure 3: EASR versus P for various ρ.

The End

Thanks for your attentions

This is the end of today's demonstration